ANNALS of the ORADEA UNIVERSITY.

Fascicle of Management and Technological Engineering, Volume VI (XVI), 2007

SOME CALCULUS MODELS APPLIED TO BRAIDED ARTIFICIAL PNEUMATIC MUSCLES

Liliana DRAGAN
North University of Baia-Mare
e-mail: lilianaubm@email.ro

Key words: Pneumatic muscle, thermodynamic calculus, geometric modelling

ABSTRACT: Braided muscle are fluidic muscles operating with compressed air, at different feeding pressures, depending on which different values of the contractile force and of their shortening degree are obtained. The aim of the paper is to present a thermodynamic calculus of braided muscle in the first part and their geometric calculus in the second part. Starting from the basic equation of the perfect gases and from the first principle of thermodynamics in the paper the equations that characterize the state transformations occurring at the braided muscle level in two situations are established: in case of some constant air masses and secondly when the masses of air that enter and get out of the muscles are variable. Then the braided muscle geometry is analyzed and some models of the fiber net that wraps up the tube membrane double-helically are achieved. Considering the muscle as a closed flexible tube, subject to inner pressure, the relation between the generated tensile force and: the feeding pressure on one side, and the muscle geometric parameters on the other side.

1. INTODUCTION

Braided muscles are linear motion engines operated by air pressure. Their essential element is represented by a flexible closed membrane, reinforced with a helical net of inextensible fibers (Figure 1).

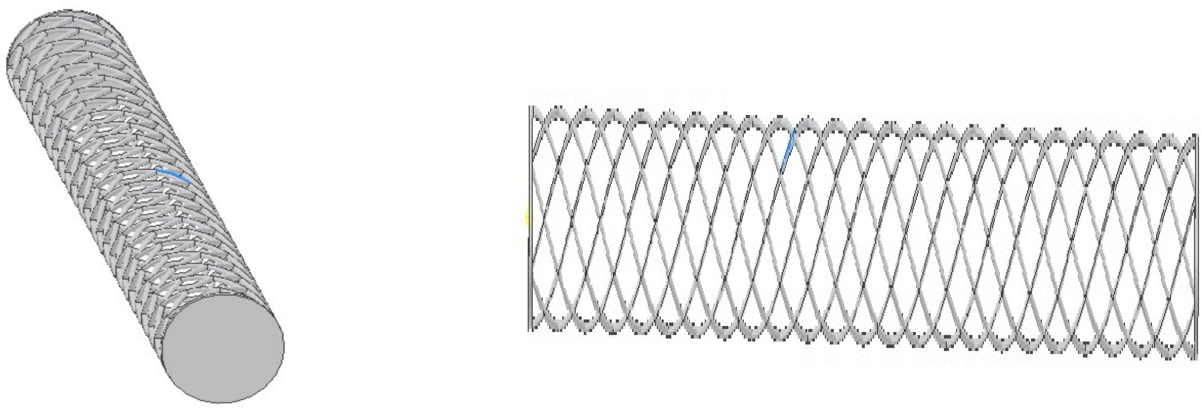

Fig.1 *Tube membrane wrapped in helical net*

Under the action of the compressed air introduced inside, braided muscle expands radially and contracts axially, producing the tensile forces.

2. THERMODYNAMIC CALCULUS OF BRAIDED MUSCLES

In order to study the processes taking place at the level of the braided artificial pneumatic muscle it is considered that the compressed air contained by it (by the muscle) is a thermodynamic system characterized by thermal parameters: volume V, pressure p, temperature T, mass m and caloric values: internal energy U, enthalpy I, entropy S.

In case of a thermodynamic system that shifts from state 1 to state 2, the heat exchange $Q_{1,2}$ and the mechanical work exchange $L_{1,2}$ can be written [3] as:

$$U_2 - U_1 = Q_{1,2} - A \cdot L_{1,2} \tag{1}$$

or as differential form:

$$du = dq - A \cdot dl \tag{2}$$

A= caloric equivalent of the mechanical work; A=43,506 kcal/Nm.
As far as the enthalpy of the system is concerned it is valid:

$$di = dq + A \cdot v \cdot dp \tag{3}$$

Figure 2 represents the artificial muscle seen as thermodynamic system.

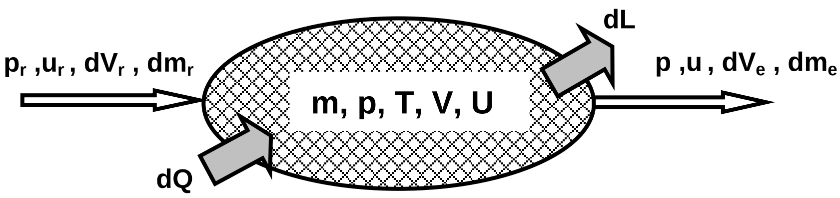

Fig.2 *Thermodynamic model of braided muscle*

In a period *dt*, in the muscle enter, from the source of compressed air, a quantity of air dm_r under the pressure p_r si, and at the same time is evacuated a quantity of air dm_e, at the pressure of the air from the muscle *p.* The air in the muscle exchanges heat with the surroundings (intake) and exchanges mechanical work (release) *dL*, by means of the end fittings.

By using the first law of thermodynamics the general equation of energetic balance of thermodynamic system is written:

$$\underbrace{\underbrace{dQ}_{\substack{caldura\\primita}} + \underbrace{u_r \cdot dm_r}_{\substack{energia\ \text{int}\ erna\\ aaeruluicare\ \text{int}\ ra\\ inMAP}} + \underbrace{p_r \cdot dV_r}_{\substack{lucrulmecanic\\ produsdeaerul\\ care\ \text{int}\ ra}}} = \underbrace{dU}_{\substack{\text{var}\ iatia\\ energiei\\ \text{int}\ ernea\\ sistemului}} + \underbrace{\underbrace{u \cdot dm_e}_{\substack{energia\ \text{int}\ erna\\ aaeruluicare\\ iese}} + \underbrace{p \cdot dV_e}_{\substack{lucrulmecanic\\ produsdeaerul\\ careiese}} + \underbrace{A \cdot p \cdot dV}_{\substack{lucrulmecanic\\ produsdeMAP}}} \tag{4}$$

Where the left term refers to the energy received by the system in the period *dt* and the right one refers to the energy released in the same period *dt* plus (+) the variation of the system internal energy. The relation (4) can also be written as:

$$dQ + i_r \cdot dm_r = dU + i \cdot dm_e + A \cdot p \cdot dV \tag{5}$$

Two situations are analyzed next:

2.1 Constant air mass

In this situation, by applying the differential of the terms that refer to the internal energy of the air from the muscle and to its volume:

$$dU = u\ dm + m\ du \tag{6}$$
$$dV = v\ dm + m\ dv \tag{7}$$

And taking into account $dm = dm_r - dm_e$ one can write:

$$dq + i_r \cdot \frac{dm_r}{m} = u \cdot \frac{dm_r}{m} - u \cdot \frac{dm_e}{m} + du + i \cdot \frac{dm_e}{m} + A \cdot p \cdot v \cdot \frac{dm_r}{m} - A \cdot p \cdot v \cdot \frac{dm_e}{m} + A \cdot p \cdot dv \tag{8}$$

The quantity of transferred heat, for the mass unit *dq*, is determined from the calorimetric relation:

$$dq = c\, dT \tag{9}$$

For the perfect gases the fundamental next state equation is:

$$pV = mRT \tag{10}$$

which, together with:

$$c_p - c_v = AR \tag{11}$$

$$\frac{c_p}{c_v} = \chi \tag{12}$$

lead to the equation that describes the transformation of a constant mass of air:

$$dq + (i_r - i) \cdot \frac{dm_r}{m} = du + A \cdot p \cdot dv \tag{13}$$

2.2 Variable masses

In the actual situation of operation of braided muscle the air masses that enter and exit the muscles are variable and this influences the parameters. In this case the relations below are valid:

$$(c_p - c_n) / (c_v - c_n) = n \tag{14}$$

$$m \cdot c_v \cdot \frac{n-\chi}{n-1} \cdot dT + c_p \cdot T_r \cdot dm_r = c_v \cdot m \cdot dT + c_V \cdot T \cdot dm + c_p \cdot T \cdot dm_r - c_p \cdot T \cdot dm + A \cdot p \cdot dV \tag{15}$$

From (10) together with c_p and c_v introduced in (15) one reaches:

$$V \cdot dp + n \cdot p \cdot dV = n \cdot R \cdot T \cdot dm + \frac{\chi(n-1)}{\chi - 1} \cdot (T_r - T) \cdot R \cdot dm_r \tag{16}$$

The relation (16) is valid in case of a polytropic transformation. But if it is considered that the operation process of braided muscle is a rapid one, where the heat exchange is negligible, then:

$$V \cdot dp + \chi \cdot p \cdot dV = \chi \cdot T_r \cdot R \cdot dm_r - \chi \cdot T \cdot R \cdot dm_e \Rightarrow dp = \frac{\chi}{V}\left(R \cdot T_r \cdot dm_r - R \cdot T \cdot dm_e - p \cdot dV\right) \tag{17}$$

The relation (17) is valid for an adiabatic transformation, when $n=\chi$.
The equations (16) and (17) can also be put in another way [2], when moreover it is considered that also $T_r=T$.

$$dp = \frac{\chi}{V}\left(R \cdot T \cdot dm - p \cdot dV\right) \tag{18}$$

The relation (18) characterizes an isentropic transformation.

In conclusion, the equations that describe the variation of the pressure, volume and air mass at the level braided muscle, in a *dt* interval, can be written as derivative form:

$$\frac{\dot{p}}{p}+\chi\cdot\frac{\dot{V}}{V}=\chi\cdot\frac{T_r}{T}\cdot\frac{\dot{m}_r}{m}-\chi\cdot\frac{\dot{m}_e}{m}$$ at adiabatic transformation

$$\frac{\dot{p}}{p}+\chi\cdot\frac{\dot{V}}{V}=\chi\cdot\frac{\dot{m}}{m}$$ at isentropic transformation

$$\frac{\dot{p}}{p}+n\cdot\frac{\dot{V}}{V}=n\cdot\frac{T_r}{T}\cdot\frac{\dot{m}_r}{m}-n\cdot\frac{\dot{m}_e}{m}$$ at polytropic transformation.

Moreover, the mass flow rate of the air that passes through a local resistance and reaches the muscles is given by the equation:

$$\dot{G}=\frac{dG}{dt}=\frac{dm}{dt}g=\dot{m}g=a\sqrt{\frac{2\cdot g\cdot\chi}{\chi-1}\cdot p_0\cdot\gamma_0\cdot\left[\left(\frac{p}{p_0}\right)^{\frac{2}{n}}-\left(\frac{p}{p_0}\right)^{\frac{n+1}{n}}\right]} \tag{19}$$

where the air flow passing through the air-nozzle, $\dot{m}$, depends on the flow area of the air-nozzle, *a,* on the upstream air pressure and on the density of the source, p_0 and γ_0, on the upstream pressure *p,* and on the polytropic exponent *n.*

3. CALCULUS OF THE PULLING FORCE

The generation of the pulling force applied to the load is closely related to the amount of air pressure and to the geometric characteristics of the muscle at rest (neither pressurized nor loaded): the initial length of the tube membrane, l_0; the initial radius (diameter), r_0 (d_0); the initial weave angle, α_0 (Figure 3).

The angle α formed by the longitudinal network fibers get modified during the functioning and the length *l*, the diameter *d* and the volume *V* of the muscle are changed as well.

When feeding with the pressure *p* which acts over the inner surface *S,* the elastic tube expands modifying its radius with *dr* and becoming shorter with *dl* (Figure 4).

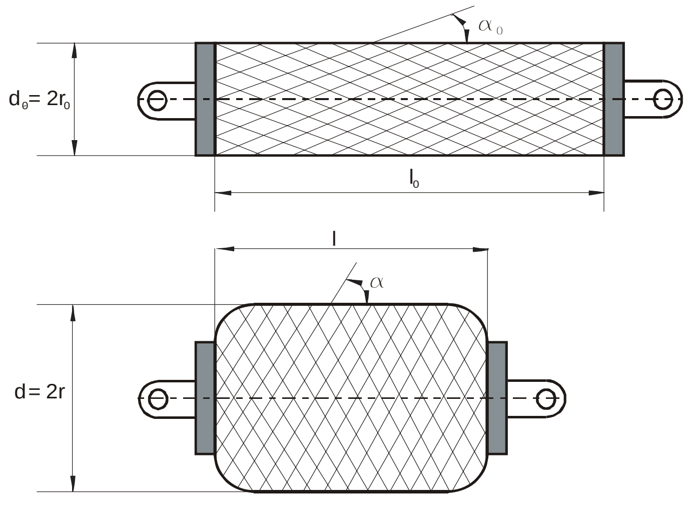

Fig.3 *Modifying the network angles*

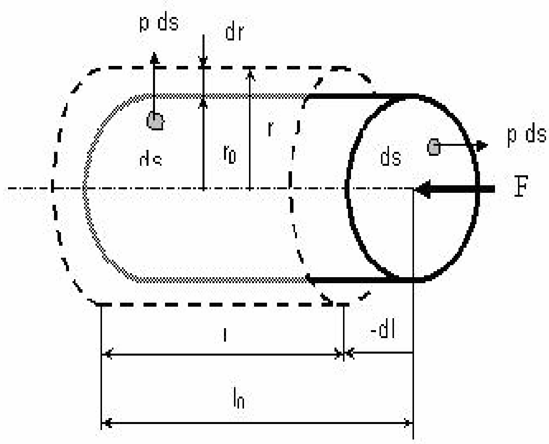

Fig.4 *Geometric modeling of the muscle expansion*

In case of balance the virtual mechanical work due to the pressure *p* (inlet) is equal with the virtual mechanical work of the contraction force (outlet), one can write successively:

$$dL_i = dL_e$$
$$dL_i = \int_S p ds \cdot dl = p \int_S ds \cdot dl = p \cdot dV$$
$$dL_e = F \cdot (-dl) = -F \cdot dl$$
$$F = -p \cdot \frac{dV}{dl} \tag{20}$$

p- relative muscle pressure; dV- variation of the muscle volume, achieved on two perpendicular directions: radial and axial.

The relation (20) is valid when the following are neglected: friction of any kind, mechanical work of deformation of the rubber tube and the inertia forces.

The following notation are made:

$$z = \frac{l_0 - l}{l_0} = 1 - \frac{l}{l_0} \text{ and } a = \frac{3}{tg^2 \alpha_0};\ b = \frac{1}{\sin^2 \alpha_0}$$

z- muscle contraction.
With these, the expression of the tensile force becomes:

$$F(p,z) = p \cdot \pi \cdot r_0^{\ 2} \cdot \left[a(1-z)^2 - b\right] \tag{21}$$

4. CALCULUS OF THE MUSCLE VOLUME

In expanded state (Figure 5), the muscle can be considered as being a cylinder body having a radius *r = d / 2* and the lengths *l*, and having two junction zones as spherical calotte. The muscle volume V_m:

$$V_m = V_0 + 2 \cdot V_c \tag{22}$$

V_0 – volume of cylindrical part; V_c – volumes of the zones at the end.

Fig.5 *Expanded muscle shape*

V_0 and V_c are calculated according to the geometric characteristics of the body.

$$V_0 = \frac{\pi \cdot d^2 \cdot (l - 2 \cdot h)}{4} \tag{23}$$

$$V_c = \frac{\pi \cdot h}{6} \cdot \left(3 \cdot r_0^2 + 3 \cdot r^2 + h^2\right) \tag{24}$$

Both V_0 and V_c can be expresed depended on the contraction z, V_0 (z) and V_c (z), and together with:

$$\begin{gathered} l = l_0 \cdot (1 - z) \\ h = \sqrt{\frac{d^2}{4} - \frac{d_0^4}{4}} \end{gathered} \tag{25}$$

on obtein the volume of the muscle calculated only dependent on the geometric elements of rest: l_0, d_0, α_0 and on the degree of shrinkage z.

$$V_0(z) = \frac{\pi}{4} \cdot d_0^2 \cdot \frac{1-(1-z)^2 \cos^2\alpha_0}{1-\cos^2\alpha_0} \cdot \left[l_0(1-z) - d_0\sqrt{\frac{1-(1-z)^2\cos^2\alpha_0}{1-\cos^2\alpha_0} - 1} \right] \tag{26}$$

$$\begin{aligned} V_c(z) = {} & \frac{\pi}{6} \cdot \sqrt{\frac{d_0^2}{4} \cdot \frac{1-(1-z)^2\cos^2\alpha_0}{1-\cos^2\alpha_0} - \frac{d_0^2}{4}} \cdot \\ & \cdot \left\{ 3 \cdot \frac{d_0^2}{4} + 3 \cdot \frac{d_0^2}{4} \cdot \frac{1-(1-z)^2\cos^2\alpha_0}{1-\cos^2\alpha_0} + \left[\frac{d_0^2}{4} \cdot \frac{1-(1-z)^2\cos^2\alpha_0}{1-\cos^2\alpha_0} - \frac{d_0^2}{4} \right] \right\} \end{aligned} \tag{27}$$

CONCLUSIONS

Braided muscle are lineal and unidirectional actuators whose source of energy is air under pressure. The operation of braided muscle is somehow similar to that of a pneumatic cylinder with simple action and spring restitution, but which has a variable area of the piston.

Since the tensile force in ideal conditions $F = -pdV/dl$, as it was noticed, this means that in case of the muscle one can speak about an “effective area” equal to –dV/dl.
Further comparing the two types of actuators: pneumatic artificial muscles and pneumatic cylinders one has found out that in case of equal feeding pressures and identical radial dimensions (diameters), the forces developed by the muscles are ten times bigger than in case of cylinders but do not remain constant over the whole duration of the stroke and decrease with it. Several relations have been deduced in the paper between pressure, volume and tensile force and the research will be developed in future.

BIBLIOGRAPHY

1. Albienz, J., Dillmann, R., Kerscher,T., Zollner, J.M. (2005) *Dynamic Modelling of Fluidic Muscles using Quick-Release*, Forschungszentrum Informatik, Germany.
2. Daerden, F. (1999) *Conception and Realization of Pleated Pneumatic Artificial Muscles and their Use as Compliant Actuation Element,* PhD Thesis, Brussel
3. Demetrescu, Th., Cosoroaba, V., (1971) *Actionari pneumatice*, E.T.,Bucuresti
4. Leonachescu, N. (1974) *Termotehnica,* EDP, Bucuresti